

Tarea 7 Unidad 2

76. Suponga que $A = i + 3j - 2k$ y $B = 4i - 2j + 4k$. Calcule a) $A \cdot B$, b) $|A|$, c) $|B|$, d) $|3A + 2B|$, e) $(2A + B) \cdot (A - 2B)$.

$$A = i + 3j - 2k$$

$$B = 4i - 2j + 4k$$

$$\mathbf{a) } A \cdot B = (i + 3j - 2k)(4i - 2j + 4k) = (4)(1) - (3)(2) - (4)(2) = 4 - 6 - 8 = -10$$

$$\mathbf{a) } A \cdot B = ?$$

$$\mathbf{b) } |A| = \sqrt{i^2 + j^2 + k^2} = \sqrt{(1)^2 + (3)^2 + (-2)^2} = \sqrt{1 + 9 + 4} = \sqrt{14}$$

$$\mathbf{b) } |A| = ?$$

$$\mathbf{c) } |B| = ?$$

$$\mathbf{c) } |B| = \sqrt{i^2 + j^2 + k^2} = \sqrt{(4)^2 + (-2)^2 + (4)^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

$$\mathbf{d) } |3A + 2B| = ?$$

$$\mathbf{d) } |3A + 2B| = |3(i + 3j - 2k) + 2(4i - 2j + 4k)| = |3i + 9j - 6k + 8i - 4j + 8k| = |11i + 5j + 2k| = \sqrt{(11)^2 + (5)^2 + (2)^2} = \sqrt{121 + 25 + 4} = \sqrt{150} = \sqrt{25 \cdot 6} = 5\sqrt{6}$$

$$\mathbf{e) } (2A + B) \cdot (A - 2B) = ?$$

$$\mathbf{e) } (2A + B) \cdot (A - 2B) = [2(i + 3j - 2k) + 4i - 2j + 4k] \cdot [i + 3j - 2k - 2(4i - 2j + 4k)] = [2i + 6j - 4k + 4i - 2j + 4k] \cdot [i + 3j - 2k - 8i + 4j - 8k] = (6i + 4j) \cdot (-7i + 7j - 10k) = (6)(-7) + (44)(7) + (-10)(0) = -42 + 28 = -14$$

77. Encuentre el ángulo entre: a) $A = 3i + 2j - 6k$ y $B = 4i - 3j + k$; b) $C = 4i - 2j + 4k$ y $D = 3i - 6j - 2k$.

$$\mathbf{a) } A = 3i + 2j - 6k$$

$$B = 4i - 3j + k$$

$$\theta = ?$$

$$\mathbf{a) } A \cdot B = |A||B| \cos \theta$$

$$(3)(4) - (2)(3) - (6)(1) = (\sqrt{(3)^2 + (2)^2 + (-6)^2})(\sqrt{(4)^2 + (-3)^2 + (1)^2}) \cos \theta$$

$$12 - 6 - 6 = (\sqrt{(3)^2 + (2)^2 + (-6)^2})(\sqrt{(4)^2 + (-3)^2 + (1)^2}) \cos \theta$$

$$0 = (\sqrt{(3)^2 + (2)^2 + (-6)^2})(\sqrt{(4)^2 + (-3)^2 + (1)^2}) \cos \theta$$

Como $A \cdot B = 0$ entonces son perpendiculares, por lo tanto su ángulo es de 90°

$$\mathbf{b) } C = 4i - 2j + 4k$$

$$D = 3i - 6j - 2k$$

$$\theta = ?$$

$$\mathbf{b) } C \cdot D = |C||D| \cos \theta$$

$$(4)(3) + (-2)(-6) + (4)(-2) =$$

$$(\sqrt{(4)^2 + (-2)^2 + (4)^2})(\sqrt{(3)^2 + (-6)^2 + (-2)^2}) \cos \theta$$

$$12 + 12 - 8 = \sqrt{36} \cdot \sqrt{49} \cos \theta$$

$$16 = 6 \cdot 7 \cos \theta$$

$$\frac{16}{42} = \cos \theta$$

$$\theta = \cos^{-1} \frac{16}{42}$$

$$\theta = 67.607$$

Tarea 7 Unidad 2

78. Calcule los valores de a para los cuales los vectores A y B son perpendiculares,

donde: a) $A = ai - 2j + k$ y $B = 2ai + aj - 4k$, b) $A = 2i + j + ak$ y $B = 2i + aj + k$.

$$\text{a) } A \cdot B = 0$$

$$\text{a) } A = ai - 2j + k \\ B = 2ai + aj - 4k$$

$$A \cdot B = 0$$

$$A \cdot B = (ai - 2j + k)(2ai + aj - 4k) = 0$$

$$A \cdot B = (a)(2a) + (-2)(a) + (-4)(1) = 0$$

$$A \cdot B = 2a^2 - 2a - 4 = 0$$

$$A \cdot B = (2a + 2)(a - 2) = 0$$

$$2a + 2 = 0 \quad a - 2 = 0$$

$$\text{b) } A \cdot B = 0$$

$$\text{b) } A = 2i + j + ak$$

$$B = 2i + aj + k$$

$$A \cdot B = 0$$

$$A \cdot B = (2i + j + ak)(2i + aj + k) = 0$$

$$A \cdot B = (2)(2) + (1)(a) + (a)(1) = 0$$

$$A \cdot B = 4 + a + a = 0$$

$$2a = -4$$